

## INTRODUCTION

Ral Rherman seeks to advise Range Resources Corporation (RRC) on the value and timing of switching from natural gas (NG) drilling in Appalachia to wet gas (NGL), as and when composite prices for wet gas exceed natural gas enough to justify switching, given the anticipated drilling costs. Range owns approximately 875,000 net acres in Pennsylvania, targeting the Upper Devonian, Marcellus and Utica/UPP shales, “stacked” in that order, allowing for multiple development opportunities. The November 2018 presentation reported that the “resource potential” (not including the proven undeveloped) of Marcellus is around 67 trillion cubic feet equivalent (Tcfe), not including Deep Utica wells or Upper Devonian, which “provide additional wet/dry optionality in the future”. There are some 3800 undrilled core wells with #300 wells 40+ Bcfe, #400 wells 30-40 Bcfe, #1400 wells 20-30 Bcfe and #1400 wells 15-20 Bcfe, (#300 wells not shown). [Multiplying the # wells shown times the Bcfe results in a total of 82 Tcfe.] Note (“SEC”) proven reserves disclosed in the 10K 2017 were 15.3 Tcfe, (6.4 proven undeveloped).

---

<sup>1</sup> © Dean A. Paxson, 2020. Parts of this case are from Valeryie Sherman, AMBS M.Sc. Finance dissertation, RO Projects at AMBS (leader Mauro Zanoletti) and ISEG (leader Alexia Dagorn) 2018, the RRC 2017 10K and November Goldman Sachs Presentation 2018, but the character is fictitious. This case is not intended as an illustration of either good or bad business practices, and mixes hypothetical and actual data and names.

The switching option evaluation considers the opportunity to shift production focus from natural gas, the traditional RRC activity in PA over the last decade, to NGL.

### Dry Gas vs. Wet Gas Prices

NGLs are often viewed as a percentage of oil prices, and are typically a multiple of dry gas.

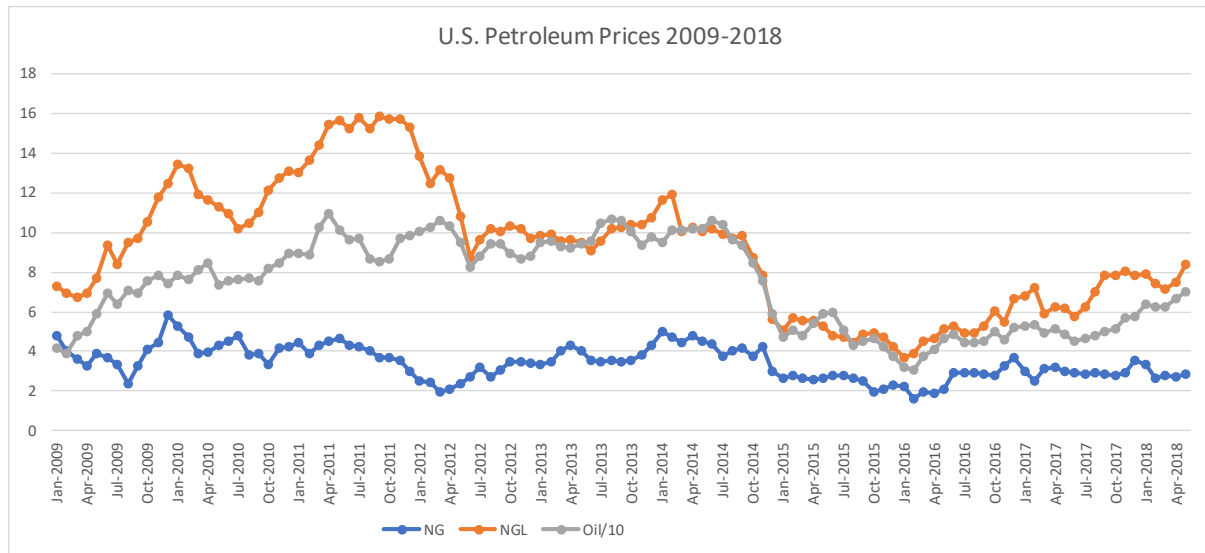


Figure 1. NGL Prices Compared to Natural Gas (E.I.A., 2018)

### Switching Model

RRC can choose to produce one of two different outputs by switching once between drilling for NG or NGL. Dockendorf and Paxson (2013) assume that the prices of the two commodity outputs,  $x=NG$   $y=NGL$ , are uncertain, possibly correlated and follow geometric Brownian motion (gBm):

$$dx = (\mu_x - \delta_x) x dt + \sigma_x x dz_x \tag{1}$$

$$dy = (\mu_y - \delta_y) y dt + \sigma_y y dz_y \tag{2}$$

with the notations:  $\mu$  expected drift of the output price,  $\delta$  convenience yield of the output,  $\sigma$  volatility of the output,  $\rho$  correlation between the two output prices and  $dz$  Wiener process (stochastic element). The instantaneous cash flow in each operating mode is the respective commodity price of the output less unit operating cost, assuming production of one (equivalent) unit per annum,  $(x - c_x)$  in operating mode ‘1’ and  $(y - c_y)$  in operating mode ‘2’. The operating costs  $c_x$  and  $c_y$  are per unit produced. A switching cost of  $S$  is incurred when switching from operating mode ‘1’ to ‘2’. The appropriate discount rate is  $r$  for non-stochastic elements, such as constant operating costs. For convenience and simplicity, assume that the appropriate

discount rate for the stochastic variables is  $\delta$ , which is equal to  $r-\mu$ . Further assumptions are that the lifetime of the asset is infinite, the company is not restricted in the product mix choice because of selling commitments, and there is no competition. Moreover, the typical assumptions of real options theory apply, with interest rates, convenience yields, volatilities and correlation constant over time.

The asset value with opportunities to switch once between the two operating modes is given by the present value of perpetual cash flows in the current operating mode plus the option to switch to the alternative mode. Let  $V_1$  be the asset value in operating mode '1', producing output  $x$ , and  $V_2$  the asset value in operating mode '2', producing output  $y$  accordingly.

$$V_1(x, y) = Ax^{\beta_2} y^{\beta_1} + \frac{x}{\delta_x} - \frac{c_x}{r} \quad (3)$$

$$V_2(x, y) = Bx^{\beta_1} y^{\beta_2} + \frac{y}{\delta_y} - \frac{c_y}{r} \quad (4)$$

where  $\beta_1$  and  $\beta_2$  satisfy the characteristic root equation

$$\frac{1}{2}\sigma_x^2\beta_2(\beta_2-1) + \frac{1}{2}\sigma_y^2\beta_1(\beta_1-1) + \rho\sigma_x\sigma_y\beta_2\beta_1 + \beta_2(r-\delta_x) + \beta_1(r-\delta_y) - r = 0, \quad (5)$$

Assuming  $c_y \geq c_x$ , and/or  $x \geq y$ , the American perpetual option to switch from  $x$  to  $y$  can be determined, so we will not consider the option value in (4). The asset value  $V_1$  is given by (3) where the first part is the value of the real option ROV to switch, and the second part is the current perpetual value of producing with output  $x$ , with the characteristic root equation (5), and  $V_2$  is given by the RHS second and third terms of (4), if  $B=0$ . Since the option to switch from  $x$  to  $y$  decreases with  $x$  and increases with  $y$ ,  $\beta_2$  must be negative and  $\beta_1$  positive. A quasi-analytical solution is obtained by considering the value matching condition (6):

$$A\hat{x}^{\beta_2} \hat{y}^{\beta_1} + \frac{\hat{x}}{\delta_x} - \frac{c_x}{r} = \frac{\hat{y}}{\delta_y} - \frac{c_y}{r} - S \quad (6)$$

and the two smooth pasting conditions at the boundaries.

## Analytical Solution

From these smooth pasting conditions:

$$\hat{y}(\hat{x}) = \frac{-\beta_1\delta_y\hat{x}}{\beta_2\delta_x} \quad (7)$$

$$A = -\frac{1}{\beta_2 \delta_x \hat{x}^{\beta_2-1} \hat{y}^{\beta_1}} \quad (8)$$

$$\frac{\hat{x}}{\delta_x} \frac{\beta_1 + \beta_2 - 1}{\beta_2} + SO = 0 \quad (9)$$

Now there is a system of three equations with four unknowns,  $\hat{x}, \hat{y}, \beta_1, \beta_2$ . Assuming that production costs are the same for x and y (or have already been incorporated into the drilling analysis), power parameters are linked through the characteristic root equation, assuming:

$$\beta_1 = (1 - \phi \beta_2) \quad (10)$$

where

$$\phi = 1 + \frac{\delta_x SO}{\hat{x}} \quad (11)$$

$$Q(\beta_1) = \beta_1^2 \{a\} + \beta_1 \{b\} - \{c\} = 0 \quad (12)$$

$$\begin{aligned} a &= \left\{ \frac{1}{2} \sigma_x^2 - \rho_{xy} \sigma_x \sigma_y \phi + \frac{1}{2} \sigma_y^2 \phi^2 \right\} \\ b &= \left\{ r - \delta_x - \phi(r - \delta_y) - \frac{1}{2} \sigma_x^2 + \frac{1}{2} \sigma_y^2 \phi + \rho_{xy} \sigma_x \sigma_y \right\} \\ c &= \left\{ -\delta_y \right\} \end{aligned} \quad (13)$$

The solution to this equation is:

$$\beta_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad (14)$$

Substituting  $\beta_1$  and  $\beta_2$  into (7) and (8) yields the analytical solutions for  $\hat{y}(\hat{x})$  and ROV.

The option to switch is:

$$ROV = A x^{\beta_2} y^{\beta_1} = \frac{-\hat{x}}{\beta_2 \delta_x} \left( \frac{y}{\hat{y}} \right)^{\beta_1} \quad (15)$$

Figure 2

	A	B	C
1	Analytical Solution Output Switching		
2	INPUTS		
3	x	100	
4	y	100	
5	$\delta x$	0.04	
6	$\delta y$	0.04	
7	$\sigma x$	0.40	
8	$\sigma y$	0.30	
9	$\rho$	0.50	
10	r	0.05	
11	cx	50	
12	cy	50	
13	S	50.00	
14	OUTPUTS		
15	X	1500	(B3/B5-B11/B10)
16	Y	1500	(B4/B6-B12/B10)
17	$\beta_1(x^\wedge)$	1.434	1-B31*B18
18	$\beta_2(x^\wedge)$	-0.425	(B32/(2*B33))-SQRT(((B32/(2*B33))^2)+(2*((B6)/B33)))
19	A	9.894	(-1/(B18*B5*(B30^(B18-1))*(B20^B17)))
20	$y^\wedge$	<b>337.043</b>	<b>(-B17*B6*B30)/(B18*B5)</b>
21	ROV	1028.979	(-B3/(B18*B5))*((B4/B20)^B17)
22	VALUE	<b>2528.979</b>	<b>B21+B3/(B5)-B11/B10</b>

Figure 2 illustrates the easy solution for the single output switch model, assuming current operating costs are half of current gross revenue for each output. If  $\hat{x} = x = 100$ , with x and y having the same initial values and the same convenience yields, the asset value X and Y excluding the switching option value is identical in both operating modes when the operating cost is the same. When operating costs are 50, the asset value  $V_1$  with a continuous switching opportunity is valued at 2529 if the incumbent is  $x=100$  with a volatility of 40%. The switching option value is the difference between the asset value and the value with no switching option,  $2529-1500=1029$ . Switching to output y is justified if y increases to 337% higher than the current output y. The spread between y and  $\hat{y}$  is due to switching costs and stochastic elements, and increases with high volatilities and low correlation, following real options theory. It should be noted that changing x also changes the switching boundary  $\hat{y}$ .

### RRC Application

In order to estimate RRC's real option to switch from drilling a well for dry gas production to drilling a well aimed at producing wet gas instead, given that dry gas and wet gas prices vary over time, it is appropriate to use a single output switching option model.

The calculations for an illustrative single NG well are presented in Table 1:



well in that location is \$9.2 million, which is the investment cost. The estimated ultimate recovery of wet gas is 30.1 Bcfe. Of these 30.1 Bcfe, the recovery of NGLs totals 2.309 million bbls., the gas recovery is 13.734 Bcf, and condensate (oil) makes up .416 million bbls, which are converted to equivalent Bcfe by multiplying by the energy equivalent. The percentages were used to determine the weighted average price of wet gas, an average price of NGL is \$7.01/Mcfe, assuming WTI \$63.80 and NGL 42% of WTI. Given the weights of NGLs, natural gas and oil in the total production output, the weighted average price of wet gas is equal to \$5.56/Mcfe.

A hyperbolic rate of -0.81 equalises the total production over 20 years and the EUR given by Range Resources. Fixed and variable lease operating costs were obtained by trial and error method so that the IRR from drilling in the super-rich area equals 62%, as disclosed by RRC. The NPV of investing in a “super-rich” well is \$24.068 million.

### Real Option Model Inputs

The inputs for both cases are summarised in Table 3:

Input Parameter	Notation	Value
Output Dry Gas	$x$	\$1.044MM
Output Wet Gas	$y$	\$1.925MM
Convenience yield of natural gas	$\delta_x$	9%
Convenience yield of wet gas	$\delta_y$	8%
Volatility of dry natural gas	$\sigma_x$	44.40%
Volatility of wet gas	$\sigma_y$	26.43%
Correlation between dry gas and wet gas	$\rho$	0.828
Risk-free interest rate	$r$	10%
Operating cost for natural gas	$c_x$	0
Operating cost for dry gas	$c_y$	0
Switching cost from dry gas to wet gas	$S_{12}$	\$1MM

Table 3. Input Parameters Description

In Table 3, the estimated values of output  $x$ , dry gas, and alternative output  $y$ , wet gas, are \$1.044 million and \$1.925 million respectively, which are the net present values of the two

outputs multiplied by their convenience yields, treating the NPV as a perpetual cashflow, net of operating costs. The volatility of dry gas prices,  $\sigma_x$ , are from data on Henry Hub natural gas spot prices for each month from January 2009 to May 2018, extracted from Bloomberg™. In the case of wet gas, the monthly data on WTI spot crude oil prices and U.S. Natural Gas Liquid Composite Prices during 2009-2018 is from the EIA website, using the given weights to find the weighted average monthly returns on wet gas over the period.

Since it is assumed that there is no option to switch back to dry gas once Range starts drilling for wet gas, the asset value in operating mode '2',  $V_2(x, y)$ , simply equals the NPV resulting from operating in that mode, \$24.068 million. The real option value in this case will be nil, as the company cannot return to operating mode '1'. Here, the value of the switching boundary suggests that for the given output level of dry gas which is \$1.044 million (the NPV of dry gas times its convenience yield), it would be reasonable to switch to wet gas production once the value of wet gas,  $y$ , reaches the level of \$1.979 million, or, equivalently, when the NPV from producing wet gas increases to \$24.742 million. The current value of wet gas,  $y$ , given the current estimated wet gas price of \$5.56, is \$1.925 million, so, in order to justify the switching decision, this value has to increase by about \$54 thousand, that is by just 3%.

In contrast, the Marshallian rule instructs that switching should take place once the difference between the alternative operating mode and the asset value in the current operating mode without any option value involved exceeds the switching cost (Dockendorf and Paxson, 2013). Since \$24.068 million is almost twice as large as \$12.603 (the NPV of \$11.603 plus the switching cost of \$1) million, managers that follow the conventional Marshallian NPV rule would have decided to switch to wet gas some time ago.

Figure 3



	B	C	D
1	Analytical Solution Output Switching		
2	INPUTS		
3	x	1.044	
4	y	1.925	
5	$\delta x$	0.090	
6	$\delta y$	0.080	
7	$\sigma x$	0.444	
8	$\sigma y$	0.264	
9	$\rho$	0.828	
10	r	0.100	
11	cx	0.000	
12	cy	0.000	
13	S	1.00	
14			
15	X	11.603	(C3/C5-C11/C10)
16	Y	24.068	(C4/C6-C12/C10)
17	$x^{\wedge}$	1.044	C3
18	OUTPUTS		
19	$\phi$	1.086	$1+(C5)/(C3)*(C13)$
20	a	0.034	$0.5*(C7^2)-C9*C7*C8*(C19)+0.5*(C8^2)*(C19^2)$
21	b	-0.051	$(C10-C5)-C19*(C10-C6)-0.5*(C7^2)-0.5*(C8^2)*C19+C9*C7*C8$
22	c	-0.080	(-C6)
23	$\beta_1(x^{\wedge})$	<b>2.038</b>	$1-C19*C24$
24	$\beta_2(x^{\wedge})$	<b>-0.956</b>	$(-C21-SQRT(C21^2-4*C20*C22))/(2*C20)$
25	$y^{\wedge}$	<b>1.979</b>	$(-C23/C24)*((C6)/(C5)*C17)$
26	ROV	<b>11.474</b>	$(-C3/(C24*C5))*((C4/C25)^C23)$
27	VALUE	23.077	$C26+C3/(C5)-C11/C10$

Ral believes that the net present value method in this case underestimates the value of the flexibility of drilling a facility for producing dry gas, ignoring the existence of the embedded option to switch to wet gas, which, in fact, adds almost 100% to the asset value.

## PROJECT QUESTIONS

1. Help Ral update the single well economics from the anticipated Feb 2020 RRC presentation, and at current (mid-March) natural gas and NGL prices. What are the new NPVs of drilling for NG and NGL, revising Table 1 and 2?
2. What are the recalculated volatilities and NG/NGL correlations, based on your reasonable assumptions?
3. What is the value of the opportunity to switch from NG to NGL?
4. Propose a plausible extra valuation to the RRC PV10 (substituting the disclosed SEC PV10 for the net capitalized cost, plus other assets less liabilities as of December 2019 considering the value of this switching option, perhaps that no more than 250 switches could possibly be made each year in the future.

## **REFERENCES**

Adkins, R. and D. Paxson (2011) “Renewing Assets with Uncertain Revenues and Operating Costs”, Journal of Financial and Quantitative Analysis, 46: 785-813.

Adkins, R. and D. Paxson (2018), “Analytical Solution for the General Two-Factor Investment Model: Option Value and Derivatives”, presented at the Real Options Conference, Düsseldorf, June.

Adkins, R. and D. Paxson (2020), “Using Input-Output Switching Options”, to be presented at the Real Options Conference, Oporto, July.

Dockendorf, J. and D. Paxson (2013) “Continuous Rainbow Options on Commodity Outputs: What is the Real Value of Switching Facilities?” European Journal of Finance, 19: 645-673.